Falsifiable predictions from semiclassical quantum gravity

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Quantum gravity is studied in a semiclassical approximation and it is found that to first order in $\sqrt{\hbar G} = l_{Pl}$ the effect of quantum gravity is to make the low energy effective spacetime metric energy dependent. The diffeomorphism invariance of the semiclassical theory forbids the appearance of a preferred frame of reference, consequently the local symmetry of this energy-dependent effective metric is a non-linear realization of the Lorentz transformations, which renders the Planck energy observer independent. This gives a form of deformed or doubly special relativity (DSR), previously explored with Magueijo, called the rainbow metric. The general argument determines the sign, but not the exact coefficient of the effect. But it applies in all dimensions with and without supersymmetry, and is, at least to leading order, universal for all matter couplings.

A consequence of DSR realized with an energy dependent effective metric is a *helicity independent* energy dependence in the speed of light to first order in l_{Pl}. However, thresholds for Tev photons and GZK protons are unchanged from special relativistic predictions. These predictions of quantum gravity are falsifiable by the upcoming AUGER and GLAST experiments.

I. INTRODUCTION

Several experiments, now in progress and planned, have the capability of testing quantum theories of gravity by looking for small quantum gravity effects on the propagation of particles, amplified by cosmological travel times[1, 2]. Among them, GLAST will be able to see energy dependent corrections in the speed of light of order $l_{Pl}E$[4], while AUGER will show whether the GZK bound is present for cosmic rays[5]. These and other experiments are tests of special relativity at high enough boosts and energies to probe quantum gravity effects of order $\sqrt{\hbar G} = l_{Pl}$.

In this letter, I report predictions for these experiments coming from the quantum theory of gravity. These predictions are generic, in that they rely only on general features of gravitational theory, that are independent of dimension and the specifics of matter couplings, as well as the presence or absence of supersymmetry. They only involve calculations at semiclassical level, to leading order in l_{Pl}. From these generic assumptions it will be shown below that to order l_{Pl} the effects of quantum gravity on the propagation of matter fields can be encompassed by the substitution of the classical metric $g_{\mu\nu}$ for a frequency dependent effective metric, $g_{\mu\nu}(l_{Pl}\omega)$ of a specific form derived below. As has been shown in detail in [6, 7], the local symmetry of this effective metric is a non-linear modification of the lorentz group acting on energy-momentum eigenstates, of a form which renders the scale l_{Pl} observer independent, in the sense that all observers will agree on the frequency of a particle with

energy $\hbar l_{Pl}^{-1}$. This is a special case of deformed or doubly special relativity[8, 9], and as such it has consequences for GLAST, AUGER and other experiments, which will be described below[1].

We assume the following general features, common to general relativity[12, 13, 14] and supergravity[15, 16] in all spacetime dimensions[17]. We assume that these assumptions are physically adequate, at least in the sense of effective field theory, at the low energies where these experiments are performed.

1. The configuration space, $\mathcal{C}$, is coordinatized by a gauge field $\mathcal{A}_a^{i\,[2]}$ where i is valued in a lie algebra $\mathbf{A}$, which may be the local lorentz algebra or a subalgebra of it. The gauge field lives on a spatial manifold of dimensions $d \geq 2$, Σ.

2. The action is of the form[12]-[17]

$$\mathcal{I} = \frac{1}{\rho} \int_{\Sigma \times R} B_i \wedge F^i + \text{constraints} + \text{matter fields} \quad (1)$$

where F^i is the field strength[3] of $\mathcal{A}^i$ and B^i is a $d-2$ form valued in $\mathbf{A}$. The constraints are quadratic, non-derivative functions of B^i whose solutions are that there exist $d+1$ frame fields e_A, $A = 0, ...d$ such that $B \approx e \wedge ... \wedge e$

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[1] The conclusions of this letter appear to differ from those of calculations in loop quantum gravity, carried out at the kinematical level[10, 11]. Presumably this is because those are based on excitations of states which are not, to any order, solutions of dynamical equations or invariant under diffeomorphisms.

[2] $a = 1, ..., d$ is a spatial index.

[3] For higher dimensional supergravity theories this is extended to include $p-$form fields, see [16].

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The conjugate momentum, $\tilde{E}^{ai} = (B_i^*)^a$ hence carries the metric information in the canonical theory[4]. The Poisson brackets of the theory are then,

$$\{\mathcal{A}_a^i(x), \tilde{E}_j^b(y)\} = \rho \delta_a^b \delta_i^j \delta^d(x, y) \tag{2}$$

here ρ is a constant of dimesions L^{d-1} that depends on the case.

3. The action is invariant under spacetime diffeomorphisms, which we will assume extends at least to order l_{Pl}. This means that to this order there is no preferred time coordinate and no preferred frame fields, except for possible expectation values of physical fields.

4. At least at the semiclassical, effective field theory level, we assume it is then adequate to consider quantum states of the form of functionals on $\mathcal{C}$, invariant under diffeomorphisms of Σ, and to represent the metric information via the operator

$$\hat{\tilde{E}}_i^a(x) = -\imath\hbar\rho \frac{\delta}{\delta\mathcal{A}_a^i(x)} \tag{3}$$

We will find it sufficient to consider semiclassical states of the form[18]

$$\Psi_0[\mathcal{A}] = e^{\imath \frac{S[\mathcal{A}]}{\hbar}} \tag{4}$$

where $S[\mathcal{A}]$ is a solution to the Hamilton-Jacobi equations which follow from the effective action (1).

Before going on, we note that even though we are working with semiclassical states, the use of (3) means that the full metric is treated quantum mechanically, as in background independent formulations. The results found here could not be derived from a perturbative treatment in which only fluctuations of geometry around a fixed classical metric are treated quantum mechanically.

In the next section we give the general argument that derives a version of DSR from these assumptions. In section III we show that the result has a natural interpretation in terms of a spacetime with one more spatial dimension, corresponding to the scale probed by an observer. Section IV probes the general argument in more detail, and reveals a connection between DSR and the cosmological constant. This conclusion is supported by an algebraic argument which is reviewed in section V. Following this, in section VI the general arguments are illustrated with a detailed example, using the Ashtekar formulation. Section VII details the predictions for near future experiments that follow from the basic conclusions found.

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[4] In $3+1$ dimensions it is the densitized spatial frame field.

II. THE GENERAL ARGUMENT

A classical solution to the effective classical theory given by (1) gives a trajectory, $\mathcal{A}_a^{0\ i}(t)$ in the configuration space, $\mathcal{C}$, where t is some parameterization of the trajectory. Solutions can be found by finding a Hamilton-Jacobi functional $S[\mathcal{A}]$, which solves the appropriate Hamilton-Jacobi equations equations which follow from (1). We then have on the classical trajectory[13, 19],

$$\tilde{E}_i^{0a}(t) = \frac{1}{\rho} \frac{\delta S}{\delta\mathcal{A}_a^i}|_{\mathcal{A}=\mathcal{A}^0(t)} \tag{5}$$

The following structure can be defined for solutions to Einstein's equations defined by Hamilton-Jacobi functionals. The classical trajectory $(\mathcal{A}_i^{0i}(t), \tilde{E}_i^{0a}(t))$ can be parameterized by a time parameter proportional to the Hamilton-Jacobi functional,

$$t_S = \nu S \tag{6}$$

where ν has dimensions of length. Furthermore, as general relativity is a local theory we can write S as the integral of a density on the d dimensional spatial manifold Σ.

$$S[\mathcal{A}] = \int_\Sigma \mathcal{S}[\mathcal{A}] \tag{7}$$

On the classical spacetime there is a time coordinate T that is proportional to $\mathcal{S}[\mathcal{A}]$. This defines a slicing of the spacetime given by the classical trajectory in which $\mathcal{S}[\mathcal{A}]$ is constant. Variations of functions on configuration space, evaluated at the classical trajectory, are then related to variations on the spacetime by,

$$\frac{d}{dT} = \mu \frac{\delta}{\delta\mathcal{S}[\mathcal{A}]} \tag{8}$$

where μ has dimensions $(length)^{-1}$.

The trajectory $(\mathcal{A}_i^{0i}(t), \tilde{E}_i^{0a}(t))$ defines a metric on a $d+1$ dimensional spacetime $\mathcal{M} = \Sigma \times R$, which is $g_{\mu\nu}$. The metric can be written as

$$\mathbf{g} = -dT^2 + \sum_i \mathbf{e}_i^0 \otimes \mathbf{e}_i^0 \tag{9}$$

where e_a^{0i} are the one form frame fields related to $\tilde{E}_i^{0a}$ on the classical trajectory.

We will also have to consider variations of $\mathcal{A}$ in the neighborhood of the classical trajectory, $\mathcal{A}^0$. These can be parameterized as[19, 20]

$$\frac{\delta}{\delta\mathcal{A}_a^i(x)} = \frac{1}{M} \tilde{E}_{i0}^a \frac{\delta}{\delta\mathcal{S}} + \frac{\delta}{\delta a_a^i} \tag{10}$$

where $\tilde{E}_0^{ai}\delta a_{ai} = 0$. The a_{ai} contain the gravitational degrees of freedom, while the trace term proportional to $\tilde{E}_{i0}^a$ can be understood as variation in the internal time coordinate. M is there to preserve the dimensions as frame

fields and metrics are dimensionless, hence its dimensions are $(length)^{d-1}$.

We can now construct a semiclassical quantum state which is a functional on the configuration space of the form (4). In the connection representation where states are functionals of $\mathcal{A}_a^i$, the operator for the densitized frame field is (3).

We now introduce matter fields, which we denote generically by ϕ. We study semiclassical quantum gravity effects on the propagation of the matter field, by considering quantum states of the Born-Oppenheimer form[18, 19]

$$\Psi[\mathcal{A}, \phi] = \Psi_0[\mathcal{A}]\chi[\mathcal{A}, \phi] \qquad (11)$$

We now consider the operator for the densitized inverse frame field acting on such states. By construction we have, when evaluated on the classical trajectory,

$$\hat{\tilde{E}}_i^a \Psi_0[\mathcal{A}] = \tilde{E}_i^{0a}\Psi_0[\mathcal{A}]. \qquad (12)$$

By the decomposition (10) we have in the neighborhood of the classical trajectory,

$$\chi[\mathcal{A}, \phi] = \chi[\mathcal{S}, a_{ai}, \phi] \qquad (13)$$

So that

$$\begin{aligned}
\hat{\tilde{E}}_i^a(x)\chi[\mathcal{A}, \phi] &= -\imath\hbar\rho\frac{\delta\chi[\mathcal{A}, \phi]}{\delta\mathcal{A}_a^i(x)} \qquad (14)\\
&= \left(\tilde{E}_i^{0a}\frac{\imath\hbar\rho}{M}\frac{\delta}{\delta\mathcal{S}(x)} - \imath\hbar\rho\frac{\delta}{\delta a_{ai}(x)}\right)\chi[\mathcal{S}, a_{ai}, \phi]
\end{aligned}$$

But, by (8) we have

$$\frac{\imath\hbar\rho}{M}\frac{\delta}{\delta\mathcal{S}(x)} = \frac{\imath\hbar\rho}{M\mu}\frac{d}{dT} \qquad (15)$$

Dimensionally, $\frac{\hbar\rho}{M\mu}$ is a time. There is only one time in the problem, which is the Planck time, so we must have

$$\frac{\hbar\rho}{M\mu} = \alpha l_{Pl} \qquad (16)$$

where α is a constant that cannot be determined at this semiclassical level of analysis. However we know that α must be finite and non-vanishing as it gives the relationship between a parameter on a configuration space trajectories and a time coordinate on the spacetime defined by that trajectory.

On the classical trajectory, we can write $\chi[\mathcal{S}, a_{ai}, \phi] = \chi[T, a_{ai}, \phi]$. Furthermore, at the semiclassical level we can neglect terms in $\frac{\delta}{\delta a_{ai}}$, which will describe couplings of matter to gravitons. We then have, neglecting graviton couplings,

$$\hat{\tilde{E}}_i^a(x)\Psi[\mathcal{A}, \phi] = \Psi_0[\mathcal{A}]\tilde{E}_i^{0a}\left(1 - \imath\alpha l_{Pl}\frac{d}{dT}\right)\chi[T, a_{ai}, \phi] \qquad (17)$$

Let us now consider a semiclassical state of definite frequency in terms of the time seen by classical observers in the spacetime, T. This must have the form

$$\chi[T, a_{ai}, \phi] = e^{-\imath\omega T}\chi_\omega[a_{ai}, \phi] \qquad (18)$$

This will be justified in section IV below.

So we have, evaluating the action of $\hat{\tilde{E}}_i^a(x)$ on a point on the classical trajectory in $\mathcal{C}$,

$$\hat{\tilde{E}}_i^a(x)\Psi[\mathcal{A}, \phi] = \Psi_0[\mathcal{A}]\tilde{E}_i^{0a}\left(1 - \alpha l_{Pl}\omega\right)\chi_\omega[T, a_{ai}, \phi] \qquad (19)$$

So we see that the effect of quantum corrections to first order in l_{Pl} is just to substitute the inverse frame field $\tilde{E}_i^{0a}$ for a *frequency dependent effective frame field*

$$\tilde{E}_i^{0a}(x, T) \rightarrow \tilde{E}_i^{0a}(x, T, \omega) = \tilde{E}_i^{0a}(x, T)(1 - \alpha l_{Pl}\omega) \quad (20)$$

As a consequence, the spacetime metric is replaced by an effective frequency dependent metric[6, 7][5].

$$g \rightarrow g(\omega) = -dT \otimes dT + \sum_i e_i \otimes e_i(1 - \alpha l_{Pl}\omega) \quad (21)$$

This leads to a universal modification in dispersion relations.

$$m^2 = -g(\omega)^{\mu\nu}k_\mu k_n u = \omega^2 - \frac{k_i^2}{(1 - \alpha l_{Pl}\omega)} \qquad (22)$$

where the one form $k_\mu = (\omega, k_i)$ contains the observed frequencies and wavevectors of physical quanta.

One can ask whether this modification in energy momentum relations corresponds to a breaking or a deformation of Lorentz invariance. To argue that it must be the latter, we recall that we assumed that the low energy effective theory is general relativity and that its gauge invariance, which is spacetime diffeomorphism invariance, holds at least to leading order in l_{Pl}. This means there cannot be an explicit breaking, from the existence of a preferred frame. The possibility that there is spontaneous breaking from some vector field getting a non-zero expectation value is eliminated by noting that the effect is universal, independent of the matter content. Among the gravitational fields which must be there, there is no suitable vector field. Hence the modified dispersion relations (22) cannot be a consequence of there being a preferred frame of reference,

The only other option is that the modified dispersion relations reflect non-linear modifications in the action of the Lorentz transformations on physical states of the matter fields. We then arrive at the conclusion that given the very mild assumptions made here, quantum gravity predicts a deformed realization of special relativity[8, 9] in the semiclassical limit. In fact, an energy dependent effective metric as in (21) is one way to express such a theory as shown in [6, 7].

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[5] in $d = 3$.

III. FIVE DIMENSIONAL INTERPRETATION

It is interesting to note that the frequency dependent effective metric $\mathbf{g}(\omega)$ can also be given a five dimensional interpretation. We note that the time it takes for information to be transfered between two modes, of frequency ω and frequency $\omega + d\omega$, is not less than ω^{-1}. Hence, if we work in a five dimensional effective spacetime, where events are labeled by coarse grained position, time and frequency (or scale), the effective causal structure is defined by a five dimensional effective metric, given by

$$ds_5^2 = -dT \otimes dT + e \otimes e(1 - \alpha l_{Pl}\omega)^2 + \frac{d\omega^2}{\omega^2} \qquad (23)$$

The "brane" at $\omega = 0$ is just the low energy effective world where classical observers live. This may be related to other five dimensional interpretations of DSR[38].

IV. DETERMINING α AND THE ROLE OF THE COSMOLOGICAL CONSTANT

We can go a little further in the general case and see how α is to be determined. This will also reveal to us why it is preferable to define quantum gravity, even at the semiclassical level, with a non-zero bare cosmological constant Λ. This means that the low energy limit in flat spacetime is to be defined through a limit in which $\Lambda \to 0$.

So far we have not imposed dynamics, we merely argued that the low energy behavior would be characterized by (18). In quantum gravity, at the semiclassical level, dynamics comes from the Hamiltonian constraint (or Wheeler-deWitt equations). These will have the form

$$\hat{H}(x)\Psi[\mathcal{A}, \phi] = 0 \qquad (24)$$

where $\hat{H}$ will be taken in the form of a density of weight one, which divides into two terms.

$$\hat{H}(x) = \hat{H}^{grav} + \rho\hat{H}^{matter} \qquad (25)$$

The gravitational part is a functional of $\hat{\tilde{E}}^{ai}$ and $\hat{F}_{ab}^i$. A full definition will require a choice of regularization and operator ordering, the details of which will not concern us at the semiclassical level. We assume only that this exists, and yields to leading order, when evaluated on the classical trajectory,

$$\hat{H}(x)\Psi_0[\mathcal{A}]|_{\mathcal{A}^0} = H[\tilde{E}_0, F^0]\Psi_0[\mathcal{A}]|_{\mathcal{A}^0} = 0 \qquad (26)$$

where $H[\tilde{E}_0, F^0] = 0$ because the classical trajectory is a background that solves the low energy field equations that follow from (1). Acting on the product state $\Psi[\mathcal{A}, \phi]$ we then have, to leading order, evaluated on the classical trajectory,

$$\hat{H}(x)\Psi[\mathcal{A}, \phi] = \Psi_0[\mathcal{A}]W[\tilde{E}_0, F_{ai}^0]_{ai}\hat{\tilde{E}}^{ai}\chi[\mathcal{A}, \phi] \qquad (27)$$

where

$$W[\tilde{E}_0, F_{ai}^0]_{ai} = \frac{\delta H^{grav}}{\delta E^{ai}} \qquad (28)$$

evaluated on the classical trajectory.

As above, we neglect graviton terms, to find that

$$\hat{H}(x)\Psi[\mathcal{A}, \phi] = \Psi_0[\mathcal{A}]W[\tilde{E}_0, F_{ai}^0]_{ai}\tilde{E}_0^{ai}(-\imath\alpha l_{Pl})\frac{d\chi[T, a_{ai}, \phi]}{dT} \qquad (29)$$

We note that it must not be the case that $W[\tilde{E}_0, F_{ai}^0]_{ai}\tilde{E}^{ai} \approx H_0^{grav} = 0$. This means that H^{grav} cannot be homogeneous in $\tilde{E}^{ai}$. It is interesting to note that in some cases, including $2 + 1$ and $3 + 1$ gravity in the self-dual, Ashtekar representation, this means that the cosmological constant cannot be non-zero.

We now turn our attention to the matter term in the Hamiltonian constraint. We have to leading order, evaluated on the classical trajectory,

$$\hat{H}^{matter}\Psi_0[\mathcal{A}]\chi[\mathcal{A}, \phi] = \Psi_0[\mathcal{A}]\hat{h}_0[\tilde{E}_0, \mathcal{A}^0, \hat{\pi}, \hat{\phi}]\chi[\mathcal{A}, \phi] \qquad (30)$$

where $\hat{h}_0[\tilde{E}_0, \mathcal{A}^0, \hat{\pi}, \hat{\phi}]$ is the quantum hamiltonian density for the matter theory on the classical background given by $(\tilde{E}^0, \mathcal{A}^0)$, which is a function of the matter field operators and conjugate momenta.

When the total Hamiltonian constraint annihilates the state to the order we are working we have then

$$\imath l_{Pl}\alpha\tilde{w}^0\frac{d\chi[T, a_{ai}, \phi]}{dT} = \rho\hat{h}_0[\tilde{E}_0, \mathcal{A}^0, \hat{\pi}, \hat{\phi}]\chi[\mathcal{A}, \phi] \qquad (31)$$

where $\tilde{w}^0 = W[\tilde{E}_0, F_{ai}^0]_{ai}\tilde{E}_0^{ai}$ is a density of weight one.

To make sense of (31) we will introduce an infrared regulator by integrating over a region $\mathcal{R} \in \Sigma$ of volume $V = \int_{\mathcal{R}} \sqrt{q_0} = L^d$. We also must take into account the fact that there will be a multiplicative renormalization in going from the matter term in the hamiltonian constraint of the quantum gravity theory defined at the Planck scale by $\hat{H}^{matter}$ and the effective low energy hamiltonian that acts in the quantum field theory defined on the classical background, $(\tilde{E}^0, \mathcal{A}^0)$. We call the latter $\hat{h}_R$ and define it by

$$\hat{h}_R = Z\hat{h}_0[\tilde{E}_0, \mathcal{A}^0, \hat{\pi}, \hat{\phi}] \qquad (32)$$

Since any sensible quantum theory of gravity must be ultraviolet finite, we expect that Z is finite in the presence of an infrared cutoff, so that

$$Z = \beta\left(\frac{L}{l_{Pl}}\right)^n \qquad (33)$$

for some power n and dimensionless constant β.

We then have

$$\imath\hbar\frac{d\chi[T, a_{ai}, \phi]}{dT}\left(\frac{\alpha l_{Pl}\int_{\mathcal{R}}\tilde{w}^0}{\rho Z}\right) = \int_{\mathcal{R}}\hat{h}_R\chi[T, \phi] \qquad (34)$$

This becomes the Schroedinger equation for quantum field theory on the background if the factor in paranthesis on the LHS is one, as we remove the infrared cutoff. This tells us that

$$\alpha = \frac{Z\rho}{l_{Pl}\int_{\mathcal{R}}\tilde{w}^0} \qquad (35)$$

In $d+1$ dimensions, $\rho = l_{Pl}^{d-1}$ is the gravitational constant. We then have roughly

$$\alpha \approx \beta\frac{L^{n-d}}{l_{Pl}^{n+1}w^0} \qquad (36)$$

In the absence of a cosmological constant, there is no scale to govern w^0, which has dimensions of l^{-2}. But if $\Lambda > 0$ we expect that the solution of interest is deSitter so that $w^0 = \eta\Lambda$, where η is another dimensionless constant of order unity. We should then simultaneously take $\Lambda \to 0$ and remove the infrared regulator, so we scale

$$\Lambda = \frac{\gamma}{L^2}\left(\frac{l_{Pl}}{L}\right)^r \qquad (37)$$

where $r > -2$ is a power that determines how the cosmological constant scales with the infrared cutoff. We have then

$$\alpha = \frac{\beta}{\eta\gamma}\left(\frac{L}{l_{Pl}}\right)^{2+n-d-r} \qquad (38)$$

which has a good limit when r is chosen so that $r = n - d + 2$. This allows us to conclude that

$$\imath\hbar\frac{d\chi[T, a_{ai}, \phi]}{dT} = \int_{\mathcal{R}}\hat{h}_R\chi[T, \phi] \qquad (39)$$

This justifies the ansatz we made (18) in the previous section.

Thus, we learn that under the same mild assumptions, we will be able to extract the quantum field theory on flat spacetime from the semiclassical approximation of the quantum gravity theory. At the same time we see what we would need to know in a concrete case to derive α and confirm it is finite. Most interestingly we see that the task of deriving quantum field theory on flat spacetime from quantum gravity is greatly facilitated if we start with the theory with a bare cosmological constant, and infrared regulator, that are scaled together as we take the limit $\Lambda \to 0$.

V. THE COSMOLOGICAL CONSTANT AND DSR

Of course it follows from all our experience with quantum field theory that we could not hope to derive the low energy limit of a quantum gravity theory otherwise than to include a bare cosmological constant. There are confirmations of this in non-perturbative approaches, such as

dynamical triangulations[39]. But in addition, there is in fact a simple algebraic argument [22] which tells us that DSR can be usefully understood in terms of the limit of quantum gravity with a cosmological constant, as $\Lambda \to 0$. The argument relies on an observation, which is that in $3+1$ dimensions, the symmetry algebra of quantum gravity with $\Lambda > 0$ is not the deSitter algebra, $SO(1, 4)$, but the quantum deformed deSitter algebra $SO_q(1, 4)$, with $q = e^{\frac{2\phi\imath}{k+2}}$ with the level k given by $k = \frac{6\pi\imath}{G\hbar\Lambda}$ [23, 24].

The limit $\Lambda \to 0$ of the quantum deformed deSitter algebra is, subject to a certain condition, not the Poincare algebra, but a quantum deformation of it called the κ−Poincare algebra. That algebra is characteristic of DSR, as discussed in [22]. The condition is that the generators of space and time translations, $\hat{P}^\mu$ emerge in the limit as

$$\hat{P}_\mu = \sqrt{\Lambda}M_{5\mu}(\frac{l_{Pl}}{L})^n \qquad (40)$$

where $M^{5\mu}$ are the dimensionless deSitter generators, $\Lambda = 1/L^2$ and the power n must be chosen so $n = 1$. (This scaling of the energy agrees with the conclusion of the previous section.) The reason we expect such a scaling is that quantum gravity coupled to matter in $3+1$ dimensions has local degrees of freedom, as well as an ultraviolet cutoff given by l_{Pl} and an infrared cutoff given by L. We note that the same argument predicts precisely the appearance of κ−Poincare as the symmetry algebra in $2+1$ dimensional quantum gravity coupled to point particles[25], although in this case $n = 0$ corresponding to the fact that this theory has no local field degrees of freedom.

VI. AN EXPLICIT EXAMPLE

We want now to combine the general semiclassical argument given in sections 2 and 4 with the algebraic argument given in the last section. To do this we discuss an explicit example involving the Ashtekar representation in $3+1$ dimensions[26, 27]. This example concerns the Kodama state([29]) and has been studied previously[13, 19]. It works both with and without supersymmetry[15]. The Kodama state is, with a particular ordering of the quantum constraints, an exact quantum state of quantum general relativity. In this paper we are, however, concerned only with its use as a semiclassical state[6].

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[6] There is, we should note, an issue about whether the full state is normalizable in the exact physical inner product, emphasized in [30]. The relevance of this issue for the present results is unclear, as we are here only concerned with it's use in a semiclassical approximation. But for interested readers, we note that the issue was studied in a linearization of quantum gravity, where it was shown that a truncation of the Kodama state is δ−functional normalizable in the Euclidean case but not in the Lorentzian

We give only the main equations here. We take for $\mathcal{A}_{ai}$ the Ashtekar connection A_a^i. The Poisson relations (2) hold with

$$\rho = \imath G \tag{41}$$

in the Lorentzian case[7]

Motivated by the arguments in sections 4 and 5, we look for DSR to emerge from a procedure in which we find quantum gravity corrections to particle propagation in deSitter spacetime, and then considering the limit as $\Lambda \to 0$. We use the fact that deSitter is the unique Lorentzian self-dual spacetime with $\Lambda > 0$. The condition of self-duality is written in Ashtekar variables as

$$F_{ab}^i = -\frac{\Lambda}{3}\epsilon_{abc}E^{ci} \tag{42}$$

In Ashtekar variables, the Hamilton-Jacobi function whose trajectories are such self-dual spacetimes is the Chern-Simons invariant of the Ashtekar connection

$$S_{CS} = \frac{2}{3\hbar G\Lambda}\int Y_{CS} \tag{43}$$

Here Y_{CS} is the Chern-Simons form, given by

$$Y_{CS} = \frac{1}{2}Tr(A \wedge dA + \frac{2}{3}A^3). \tag{44}$$

It satisfies $\frac{\delta \int Y_{CS}}{\delta A_{ai}} = 2\epsilon^{abc}F_{bc}^i$.

We will consider $\Sigma = R^3$ so $A_a^{0i}(t)$ parameterizes a flat slicing of deSitter spacetime. To define the Hamilton-Jacobi function corresponding to solutions which are homogeneous in the flat slicing, we will have to impose an infrared cutoff, as in the general case. So we studying the system on $\Sigma = T^3$ rather than R^3, with a periodicity R. We will then take $R \to \infty$ as we take $\Lambda \to 0$[8]. In a convenient gauge, the trajectory on configuration space which corresponds to the flat slicing of deSitter is[13]

$$A_{ai}^0 = \imath\sqrt{\frac{\Lambda}{3}}\,f(T)\,\delta_{ai} \;\mapsto\; F_{abi}^0 = -f^2(T)\,\frac{\Lambda}{3}\epsilon_{abi} \tag{45}$$

where $f = e^{HT}$, with $H = \sqrt{\frac{\Lambda}{3}}$. From the self-dual condition (42) we have

$$E_0^{ai} = f^2\delta^{ai} \;\mapsto\; q_{ab}^0 = f^2\delta_{ab} \tag{46}$$

In the Lorentzian case S_{CS} is complex. The global time coordinate of interest is

$$t_{CS} = \nu\mathcal{I}mS_{CS} \tag{47}$$

This has good properties for a time coordinate on the configuration space of general relativity[19]. For example, for small $\lambda = G\hbar\Lambda$ it closely approximates York time, which is a well studied instrinsic time coordinate. The local time coordinate is

$$\tau_{CS}(x) = \left(\frac{\Lambda}{3}\right)^{3/2}e^{3\sqrt{\frac{\Lambda}{3}}\tau(x)} \tag{48}$$

here $\tau(x)$ is a field which parameterizes the trace part of A_{ai}. In this case we can invert the expression for the derivatives of A_{ai} in the neighborhood of the classical solution (10), to find,

$$A_{ai}(x) = \imath\sqrt{\frac{\Lambda}{3}}\delta_{ai}e^{\sqrt{\frac{\Lambda}{3}}\tau(x)} + a_{ai} \tag{49}$$

On the solution of interest, $\tau(x) = T$.

Following the general argument we construct the semi-classical wavefunctional

$$\Psi_0(A) = \mathcal{N}e^{\frac{3}{2G\hbar\Lambda}\int Y_{CS}} \tag{50}$$

Following the logic of the general case, we have to study the action of the inverse metric operator on $\chi[A,\phi]$. We have

$$\hat{E}^{ai}\chi(A,\phi) = -\hbar G\frac{\delta\chi(A,\phi)}{\delta A_{ai}} = \frac{\imath\hbar G}{\Lambda}\delta^{ai}\frac{\delta\chi(A,\phi)}{\delta\tau} - \hbar G\frac{\delta\chi(A,\phi)}{\delta a_{ai}} \tag{51}$$

We are interested in extracting quantum field theory on Minkowski spacetime, in the limit $\Lambda \to 0$. For the limit to be non-singular we must rescale the time coordinate, because of the factors of $\hbar G/\Lambda$ in front of the $\delta/\delta\tau$ derivatives. In any case we need to rescale to remove a density factor. To do this we must replace the functional degree of freedom $\tau(x)$, which we have chosen to represent time by a global coordinate T. This coordinate T is taken to be proportional to τ on a $\tau = $ constant slice. However $\delta/\delta\tau(x)$ and $\partial/\partial T$ have different density weights and dimensions and this must be compensated for.

We accomplish both if we rescale so that on a fixed $\tau = $ constant slice,

$$\frac{\hbar G}{\Lambda}\frac{\delta}{\delta\tau(x)} = \alpha l_{Pl}\sqrt{detq_{ab}^0}\frac{\partial}{\partial T} \tag{52}$$

Thus, we arrive at (17), from which we drew our physical conclusions.

In fact, we can do much more in this explicit example. In [13, 19] we studied the specific case of a scalar field and show that the Hamiltonian constraint on the product state reduces in the limit of small λ to a Schrodinger equation plus corrections.

$$\imath\frac{\delta\chi}{\delta\tau_{CS}} = \frac{1}{\Lambda}H_{E^{ai}=(3/\Lambda)\epsilon^{abc}F_{bc}^i}^{matter}\chi + O(l_{Pl}E) \tag{53}$$

This justifies the decomposition (18). In this equation, the matter Hamiltonian is evaluated with classical gravitational fields satisfying the self-dual condition

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case [31]. It was also found that a truncation to homogeneous cosmology can be made normalizable by making the cosmological constant depend on a physical field and making wavepackets in that field's value[32]. Another approach to the issue is in [33].

[7] The Euclidean case works as well, but with $\rho = G$. For details see [13]. One can treat the AdS case as easily as the deSitter case, but in this case one should be careful about boundary conditions.

[8] See [13] for details.

$E^{ai} = (3/\Lambda)\epsilon^{abc}F^{i}_{bc}$. This justifies the choice of time co-ordinate (47).

Another feature of this time coordinate justifies its use, which is that it allows us to recover the thermal nature of quantum field theory on deSitter spacetime[28], and in fact extend it to the full quantum gravity theory[13, 19]. If we continue to the Euclidean case, the Ashtekar connection becomes real and the corresponding internal time coordinate is just

$$\tau_{ECS} = \int_{\Sigma} Y_{CS}(A) \qquad (54)$$

We can consider the effect of this on an S^3 slicinig of Euclidean deSitter spacetime. In this case the configuration space becomes periodic. Two points in the configuration space $\mathcal{C}$ which differ by a large gauge transformation with winding number n should be physically identified. This means that two points on a trajectory in $\mathcal{C}$ connected by

$$\int Y_{CS}(A) \rightarrow \int Y_{CS}(A) + 8\pi^2 n \qquad (55)$$

for any n must be identified. Hence $\tau_{ECS} = \int Y_{CS}(A)$ is actually a periodic function on the configuration space. As a result, *every* correlation function will satisfy the KMS condition in T_{ECS}, no matter what the state. That is, by equating configurations of A_{ai} that differ by a large gauge transformations we reduce the topology of the configuration space to a circle, which is parameterized by τ_{ECS}. This time coordinate is dimensionless, and by (55) its periodicity is $8\pi^2$. By evaluating this on a classical solution corresponding to Euclidean deSitter one can recover[13] the dimensional temperature of deSitter spacetime[28]

$$\mathcal{T} = \frac{1}{2\pi}\sqrt{\frac{\Lambda}{3}} \qquad (56)$$

One can also recover the entropy of deSitter spacetime, but that derivation would take us too far afield[13].

Thus, in the case of this example we see in detail that the notion of time on the configuration space of the gravitational field we discussed in the general case, based on a Hamilton-Jacobi function, indeed gives a physically meaningful time, whose use allows us to recover known facts about quantum field theory on deSitter spacetime. By imposing the physically plausible requirement that such a time coordinate be related to a time coordinate on a spacetime which is derived from that same Hamilton-Jacobi function, we arrive at the conclusions found above.

VII. APPLICATIONS AND PREDICTIONS

The conclusion of the previous sections is then that, at least to leading order in l_{Pl} the effects of quantum gravity on the propagation of matter fields is described by switching from the bare metric to the frequency dependent effective metric $\mathbf{g}(\omega)$, given by (21). The case of the electromagnetic field can serve to illustrate the consequences.

We begin with the contribution for the Hamiltonian constraint from the Maxwell field

$$H^{Maxwell} = \frac{1}{2}q_{ab}\tilde{\pi}^a\tilde{\pi}^b + \frac{1}{4}qq^{ac}q^{bd}f_{ab}f_{cd} \qquad (57)$$

where f_{ab} is the magnetic field of the vector potential a_a and $\tilde{\pi}^a$ is its conjugate momentum. To find the equations of motion we have to smear with a lapse function N of density weight minus one.

$$H^{Maxwell}(N) = \int_{\Sigma} N H^{Maxwell} \qquad (58)$$

We now act on a product state of the form (11) and take the limit to evolve around a flat spacetime background. To evolve in the time coordinate of an inertial observer we must pick the inverse densitized lapse $N = q^{-\frac{1}{2}}$. We find, using (20), the theory is equivalent at the semiclassical level to evolution by the hamiltonian

$$H^{Maxwell}(N) = \int d^3k \frac{1}{\sqrt{1-\alpha l_{Pl}\omega}} \left[\frac{1}{2}\delta_{ab}\tilde{\pi}^a(k)\tilde{\pi}^b(k)\right.$$
$$\left. + \frac{1}{4}\delta^{ac}\delta^{bd}f_{ab}(k)f_{cd}(k)\right] \qquad (59)$$

Making the usual gauge choices for the potential a_μ, of $a_0 = 0, k \cdot a = 0$ we arrive at the equations of motion for $a(k,t)_a = a_\omega(k)_a e^{-\iota\omega t}$,

$$\left(\omega^2 - \frac{k \cdot k}{1-\alpha l_{Pl}\omega}\right)a_\omega(k)_a = 0 \qquad (60)$$

Thus, we arrive at the helicity independent dispersion relations (22). This, we might note, disagrees with an analysis based on effective field theory for the case of Lorentz symmetry breaking [34]. The form of effective field theory depends on the low energy symmetry, and were it only rotations there would be a helicity dependence in the dispersion relations[34]. That such a term is absent confirms that the low energy symmetry is deformed rather than broken Lorentz symmetry[9]. A similar analysis leads quickly to the conclusion that the same helcity independent dispersion relation governs the propagation of spin-$\frac{1}{2}$ fields.

There are stringent experimental limits on first order in l_{Pl} modifications to dispersion relations in a lorentz breaking scenario[1, 2], but not in a DSR scenario[3].

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[9] Note that effective field theory plus explicit lorentz symmetry breaking has drastic consequences that disagree with experiment[35]. This strongly implies that if quantum gravity effects modify particle propagation, they do so in a way that deforms rather than breaks lorentz invariance.

There are also stringent limits on helicity dependent first order corrections to the speed of light, coming from rotations of planes of polarization[36], but these also do not apply here. We close with some observations on consequences for near future experiments.

First, there is an helicity independent energy dependent speed of light, given by[10]

$$v = c(1 + \alpha l_{Pl}\omega + ...) \qquad (61)$$

We may note that this implies that the speed of light will increase in the early universe when the temperature is high. Possible implications for early universe cosmology are reviewed in [40].

Second there are consequences for threshold experiments such as Tev photons and GZK protons. To analyze these one needs to know how to apply conservation of energy and momentum. We will assume, following [37] that while it is the covariant energy and momentum $k_\mu = (\omega, k_i)$ which are observed, it is the contravariant 4-vectors

$$k^\mu = g^{\mu\nu}(\omega)k_\nu = (\omega, \frac{k_i}{(1 - \alpha l_{Pl}\omega)})) \qquad (62)$$

which are conserved linearly.

One can then study threshold reactions and find that, as in previous analyses[3, 8, 37], the thresholds are not moved significantly.

It then appears that one can separate the three cases having to do with the fate of Lorentz invariance:

[10] Note that the sign agrees with that required for DSR, as shown in [3].

- **Ordinary Lorentz invariance** predicts normal thresholds and no energy dependence in the speed of light.

- **Explicit breaking of Lorentz invariance** predicts changes of $O(1)$ to threshold energies for Tev photons and GQK protons and a *helicity dependent* energy dependent speed of light. We note there are other predictions as well, some of which have been falsified, at least to first order in l_{Pl}.

- **DSR** predicts no changes to thresholds but an *helicity independent* first order in l_{Pl} modification of the speed of light.

Remarkably, it appears that the AUGER and GLAST experiments together could be sufficient to distinguish these three cases.

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